FACULTY DEVELOPMENT ENDOWMENT FUNDS

Application Cover

Date of Travel: 5/29/2016 -- 6/04/2016

Applying for: ☑ Eleanor Lane Endowment ☐ Nathan Deutsch Development Fund

Principal Investigator: Jeongho Ahn Dept: Mathematics and Statistics

Other Investigators: N/A

Proposal Title: Dynamic Frictional Contact of Viscoelastic Gao Beams

Have you received this funding in the past? ○ Yes ◉ No If yes, when? _____

Proposed Budget

	Funds Requested
Travel	$ 1,000
Supplies & Materials	$
Other (Specify)	$
TOTAL REQUEST	$ 1,000

Brief Abstract for Publication on RTT Website: [Attach separate page if you have difficulty with the form.]

Attached please see the separate form.

Signature of Applicant: Ahn Jeongho Date: 10/25/15

Signature of Department Chair: Debra Ingram Date: 10/25/15

Signature of Collegiate Dean: John M. Pratte Date: 10/25/2015

**Electronic signatures are acceptable. Submit electronically to research@astate.edu OR to RTT office.*

DYNAMIC FRICTIONAL CONTACT OF VISCOELASTIC GAO BEAMS

JEONGHO AHN, PH.D.

Abstract. In this work, we study the mathematical analysis and numerical schemes of dynamic frictional contact between a viscoelastic Gao beam and a stationary rigid foundation. The motion of the Gao beam is formulated by a *nonlinear* hyperbolic partial differential equation (PDE) and its contact condition satisfies Coulomb law of dry friction. Previously, the classical Finite Element Methods (FEMs) were employed to propose numerical schemes of the nonlinear beam over the spatial domain. However, they did not seem to work perfectly. In order to improve the numerical schemes, we apply $\mathcal{C}^0$ interior penalty methods ($\mathcal{C}^0$-IPMs) to the frictional contact problem of the nonlinear beam. Indeed, the $\mathcal{C}^0$-IPMs which are discontinuous Galerkin methods have been getting popular to solve fourth order PDEs. Unlike the standard linear beams such as Euler–Bernoulli beams or Timoshenko beams, the energy function of the Gao beam is non-convex. Thus, there are three critical points which are solutions of the *static* Gao beams. An important task is that we investigate advanced optimization techniques to cooperate with a time discretization. In addition, convergence of numerical trajectories to the nonlinear PDE system is proved. A fully numerical scheme which combines the $\mathcal{C}^0$-IPMs with the time discretization is implemented. We provide a numerical example in which numerical results and simulations are presented, as well.

PROPOSAL SUMMARY

Title: Dynamic Frictional Contact of Viscoelastic Gao Beams

Applicant: Jeongho Ahn, Ph.D.

Department of Mathematics & Statistics

I have been invited to participate and make a presentation in mini-symposium on "Mathematical Analysis of Unilateral Contact Problem" within an international conference. The conference whose name is Emerging Trends in Applied Mathematics and Mechanics will be held May 30 – June 3, 2016 in the University of Perpignan Via Domitia, France. Professor Meir Shillor, one of the mini-symposium organizers, is a well known applied mathematician in my research area who has been collaborating with me, since I started working as an assistant professor in A-State. The mini-symposium will be also dedicated to the memory of Dr. M. Schatzman (12/8/1949–08/20/2010) and J. J. Telega (3/24/1943–1/21/2005). They were quite excellent researchers in our research community.

There are abundant dynamic contact models of *linear* beams such as Euler–Bernoulli beams or Timoshenko beams. Those contact problems with additional effects have been studied through many papers in applied mathematics communities, using mostly variational formulation techniques and penalty methods. Many numerical schemes for those mathematical models have been proposed and developed, as well. Due to the Signorini contact conditions, the dynamic contact problems basically, become *nonlinear* hyperbolic type of partial differential equations (PDEs). One of excellent numerical approaches for the nonlinear system is to use complementarity conditions (CPs). Indeed, understanding the Signorini conditions as CPs is very useful to compute numerical solutions for thick obstacle problems, i.e., multiple contact zones on elastic or viscoelastic (Kelvin–Voigt type) bodies.

While the classical FEMs for the dynamic contact models of the linear beams work properly, they do not seem to be appropriate for highly nonlinear beams, called Gao beams, which are initially proposed by Gao in 1996. However, $\mathcal{C}^0$ interior penalty methods ($\mathcal{C}^0$-IPMs) which are discontinuous Galerkin (DG) methods have been recently proposed, improving the shortcomings of the classical approaches. Especially, my research colleague and I have recently investigated on a possibility of applying $\mathcal{C}^0$-IPMs to the nonlinear beam model and have obtained interesting results. The joint work will be submitted to a peer-reviewed journal soon. Although I published a coupled papers about dynamic contact of Gao beams, the numerical schemes proposed in the papers did not seem to be perfect. I plans to extend the nonlinear beam model to *frictional* contact problems, applying the $\mathcal{C}^0$-IPMs over the spatial domain and a time discretization over the time interval. This extended contact model is a complicated nonlinear system and thus will be a challenging project. In addition, frictional contact problems with Coulomb's law of dry friction cause inherent instability.

There still remain unsolved issues in the model of the nonlinear beams. One of them is to find the global minimum of total energy function at each time step. This may seem to be an extremely difficult task, because of the *nonconvex* energy function. Indeed, the energy function has three critical solutions in the static case: one local maximum and two local minimums. Generally speaking, there is no perfect optimization technique to compute the global minimum of nonconvex functions. A possibility of finding the global point is convexization, handling all data, the Gao coefficient, and the horizontal traction appropriately. Probably, identifying the coefficient and the traction may be a good idea to improve the dynamic model of the nonlinear beams.

This project will require more fundamental investigations on similar and simpler beam models. A undergraduate researcher who submitted the 2016 SURF proposal will have a great opportunity to contribute to my research community, since the proposal will play a significant role in proposing more efficient and accurate numerical schemes for the dynamic contact model of Gao beams. When I attend the international conference, I will have a valuable communication with Dr. Gao to combine my specialty with optimization techniques and theories developed by him.

In order to present at the international conference, France, my request for Eleanor Lane Endowment funds is a total amount $\$1,000$ which, I hope, will be used to purchase a flight ticket and the registration fee. Unfortunately, I do not know about the precise price of a flight ticket and the registration fee yet. However, I am expecting that travel, registration, and accommodation fees will cost approximately $\$2,500$. When I had a meeting with Dr. Ingram, department chair, she said that the department would also support me to give a talk at the French conference.

The expected results in my long term project will be contributed to develop interdisciplinary studies between applied mathematics and engineering. Communications with experts in the international conference will be an essential processing to continue to make my scholarly activities even wider and disseminate publicly. Indeed, many well known experts who are interested in processes involved in contact mechanics will be invited to participate in the international conference. Another important impact of my participation in the meeting is to strengthen my research mentorship to collaborate with undergraduate and graduate researchers. In fact, I have been able to find several excellent students who have a high potential to do research and are qualified for education and research endeavor of A-State. I am one of junior faculty in the department of Mathematics and Statistics who has been showing active research career. I have been a mentor for several undergraduate students who have written their honors thesis. I was also a thesis advisor of a student in our masters program, and am currently advising a master student who will defend his thesis coming spring 2016. Please see my detailed research mentorship in the Curriculum Vitae. I plan to continue and expand my scholarly activities to involve excellent and mature students in A-State.

JEONGHO AHN, PH.D. CURRICULUM VITAE

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PROFESSIONAL PREPARATION

February 1989 Kyung Hee University, Seoul Korea
B.S. in Mathematics, February 1989
February 1991 Kyung Hee University, Seoul, Korea
M.S. in Mathematics, February 1991
December 2003 The University of Iowa, Iowa City, Iowa
Ph.D. in Mathematics, December 2003

APPOINTMENTS

Fall 2015 – present Associate Professor,
Department of Mathematics & Statistics, Arkansas State University
Fall 2009 – Spring 2015 Assistant Professor,
Department of Mathematics & Statistics, Arkansas State University
Fall 2008 – Spring 2009 Visiting Assistant Professor,
Department of Mathematics & Statistics, Arkansas State University
Fall 2006 – Summer 2008 Full Time Lecturer,
Department of Mathematics & Physics, Alfred State SUNY College
Spring 2005 – Summer 2006 Post doctor and instructor,
Department of Mathematics, Yonsei University, South Korea

AREA OF RESEARCH

Applied Mathematics
Numerical Analysis
Partial Differential Equations
Dynamic Contact Problems
Finite Element Methods
Complementarity Problems
Differential Variational Inequalities

PRODUCTS

- PRODUCTS MOST CLOSELY RELATED

Peer-reviewed Journals

1. "Dynamic frictionless contact of a nonlinear beam with two stops" with Eun-Jae Park, *Applicable Analysis*, 94 (2015), pp.1355 – 1379.
2. "A viscoelastic Timoshenko beam with Coulomb law of friction", *Applied Mathematics and Computation*, 218 (2012), pp.7078 – 7099.
3. "Dynamic contact of two Gao beams" with Kenneth Kuttler and Meir Shillor, *Electronic Journal of Differential Equations*, 2012 (2012), pp.1 – 42.

- OTHER SIGNIFICANT PRODUCTS

Peer-reviewed Journals

1. "Mathematical and numerical approaches to a one-dimensional dynamic thermo-viscoelastic contact problem" with Natanya Clark, *Nonlinear Analysis: Real World Applications* 22 (2015), pp.437 – 451.
2. "Dynamic impact of a particle" with Jared R. Wolf, *Involve - a journal of mathematics*, 6 (2013), pp.147 – 167.
3. "Dynamic contact of viscoelastic bodies with two obstacles: Mathematical and numerical approaches" with Jon Calhoun, *Electronic Journal of Differential Equations*, 2013 (2013), pp.1 – 23.
4. "A viscoelastic Timoshenko Beam with dynamic frictionless impact", with David E. Stewart, *Discrete Contin. Dyn. Syst. Ser. B*,12 (2009), pp.1 – 22.
5. "Dynamic frictionless contact in linear viscoelasticity", with David E. Stewart, *IMA Journal of Numerical Analysis*, 29 (2009), pp.43 – 71
6. "Thick obstacle problems with dynamic adhesive contact", *ESAIM: Mathematical Modeling and Numerical Analysis*, 42 (2008), pp.1021 – 1045.
7. "An Euler-Bernoulli Beam with Dynamic contact: Penalty approximation and existence", with David E. Stewart, *Numerical Functional Analysis and Optimization*, 28 (2007), pp.1003 – 1026.
8. "A vibrating string with dynamic frictionless impact", *Applied Numerical Mathematics*, 57 (2007), pp.861 – 884
9. "Existence of solution for a class of impact problems without viscosity" with David E. Stewart, *SIAM J. Math. Anal.*, 38 (2006), pp.37 – 63 (electronic).
10. "An Euler-Bernoulli Beam with Dynamic Contact: Discretization, Convergence, and Numerical Results" with David E. Stewart, *SIAM J. Numer. Anal.*, 43 (2005), pp.1455 – 1480 (electronic).

Refereed Conference Paper

11. "A simplified model of impact" with David E. Stewart, Contact Mechanics: Proceedings of the Third Contact Mechanics International Symposium, *Kluwer Acad. Publ*, (2002), pp. 309 – 316.

Book (Solutions Manual)

12. "Instructor's Solutions Manual for Elementary Numerical Analysis by Kendall E. Atkinson and Weimin Han", 3rd edition, joint with Dr. David Chien and Dr. Zongmi Wu, *John Wiley & Sons, Inc.*, (350 pages).

RESEARCH SUPPORT

- CURRENT

1 "Nonlinear springs with a pair of complementarity conditions" with Jay Mayfield, in progress.
2. "Mathematical and numerical approaches to cracked Euler–Bernoulli beams" with Jay Mayfield, in preparation.

- PENDING

1 "C^0 interior penalty finite element methods for nonlinear fourth order differential equations with Seulip Lee and Eun-Jae Park, to be submitted soon.
2 "Modeling, analysis and simulations of a dynamic thermoviscoelastic rod/beam system" with Kenneth Kuttler, Meir Shillor, under review.

SYNERGISTIC ACTIVITIES

- Since I started working as a pre-tenure professor, I have been a research mentor to several undergraduate researchers and an thesis advisor to a graduate student. When I mentored them, undergraduates and a graduate received a SURF grant and an ASGC grant. respectively. Here is the list of the grants.

1. SURF (Student Undergraduate Research Fellowship) with Jared R. Wolf, Arkansas Department of Higher Education, 01/01/2010 – 12/1/2010
2. SURF with Jared R. Wolf, Arkansas Department of Higher Education, 01/01/2011 – 12/1/2011
3. ASGC (Arkansas Space Grant Consortium) with Robert Monteforte (A Graduate Student of Mathematics), "The mathematical modeling of a space elevator", 07/01/2012 – 6/31/2013
4. SURF with Natanya Clark, "Numerical Approaches to Thermoelastic Rods with Dynamic Contact", 01/01/2013 – 12/1/2013

5. SURF with Jay Mayfield, "Mathematical Approaches to Nonlinear springs with Dynamic Contact", 01/01/2014 – 8/31/2014

Two of undergraduate researchers extended their project to honors thesis, respectively. They also published their joint paper with me in the peer-reviewed journals. You can find the papers in the products. In addition, the master student extended the project to his master thesis. All the undergraduate and graduate researchers made their presentations in national or regional conferences.

- I have been collaborating my research colleagues who are well known researchers in their specialties. See the recent publications. I have been working as a mathematical reviewer for MathSciNet and a referee for many peer-reviewed journals.

- More recently, I have been advising a master student who writes his thesis. Indeed, he has continued to work on two consecutive projects: one is about nonlinear springs and the other is about cracked Euler–Bernoulli beams. We hope that two papers related to the projects will be published in peer-reviewed journals. A female undergraduate researcher submitted the 2016 SURF grant and she plans to write her honors thesis under my supervision.